

Within the framework of the linear thermodynamics of irreversible processes, a general form is obtained for the equations of isothermal nonequilibrium filtration of an incompressible liquid through a nondeforming porous matrix.

If thermodynamics is to be used to establish the form of the relation between the filtration velocity  $\mathbf{q}$  and the pressure gradient  $\nabla p$ , the energy  $\Phi$  scattered in a unit volume of porous medium in unit time must be expressed in terms of the mean parameters used in filtration theory. Consider, for simplicity, the isothermal case. The liquid is assumed to be incompressible, and the porous matrix to be nondeforming. The first and second laws of thermodynamics will be written for unit volume of the porous medium as a whole. The result, in the usual notation (see [1], pp. 207, 242), is

$$dQ^{(e)} = dU + dA^{(i)}, TdS = dQ^{(e)} + dQ'. \tag{1}$$

In order to determine the dissipative function, the Gibbs relation must also be used. It is taken in the following form

$$TdS = dU - \mathbf{B}_\alpha \cdot d\eta_\alpha. \tag{2}$$

Here and below, summation from 1 to  $n$  over the repeating indices is assumed. Equation (2) is equivalent to the assumption  $U = U(S, \eta_\alpha)$ . The internal variables  $\eta_\alpha$  on average over the volume take account of the capacity of the liquid-porous-matrix system for accumulating energy. There may be various mechanisms of this accumulation [2, 3]. The variables  $\mathbf{B}_\alpha$  are conjugate with the  $\eta_\alpha$  parameters. From Eqs. (1) and (2) it is found that

$$\Phi = \frac{dQ'}{dt} = - \frac{dA^{(i)}}{dt} - \mathbf{B}_\alpha \cdot \frac{d\eta_\alpha}{dt}.$$

Next,  $dA^{(i)}/dt$  will be expressed in terms of the averaged parameters used in filtration theory. It is known (see [1], pp. 207-208) that

$$N = \int_V - \frac{dA^{(i)}}{dt} dV = \int_V \sigma_{ij} \frac{\partial v_i}{\partial x_j} dV = \int_V \frac{\partial}{\partial x_j} (\sigma_{ij} v_i) dV - \int_V \frac{\partial \sigma_{ij}}{\partial x_j} v_i dV. \tag{3}$$

Here  $x_j$  are Cartesian coordinates;  $\sigma_{ij}$ , symmetric stress tensor in the liquid;  $v_i$ , liquid-particle velocity;  $V$ , part of the porous-medium volume  $V_0$  occupied by liquids. If inertial terms are neglected in the equations of liquid-particle motion (see [1], p. 143) and (for simplicity) so are mass forces, then  $\partial \sigma_{ij} / \partial x_j = 0$  and

$$N = \int_{S_e \cup S_i} \sigma_{ij} v_i n_j dS.$$

$S_e$  is the liquid part of the surface  $S_0$  of volume  $V_0$ ;  $S_i$  is the interphase surface inside  $V_0$ . On  $S_i$ , the condition of liquid-particle adhesion to the porous matrix is satisfied; therefore

$$N = \int_{S_e} \sigma_{ij} v_i n_j dS. \tag{4}$$

This relation permits transition to averaged parameters. In fact, since the surface area  $S_i$  is much larger than the surface area  $S_e$ , the tangential stress on  $S_e$  may be neglected in comparison with the pressure. If the pressure pulsations at  $S_e$  are also neglected, then Eq. (4) may be written in the form

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$$N = \int_{S_0} -Pq_i n_i dS_0. \quad (5)$$

Passing in Eq. (5) to the integral over the volume, and taking the continuity equation into account, it is found that

$$N = \int_{V_0} -q_i \frac{\partial P}{\partial x_i} dV_0.$$

Thus

$$-\frac{dA^{(i)}}{dt} = -\mathbf{q} \cdot \nabla P, \quad \Phi = -\frac{\partial P}{\partial x_i} q_i - \mathbf{B}_\alpha \cdot \dot{\eta}_\alpha. \quad (6)$$

To establish the filtration law corresponding to Eq. (6), Onsager's theory is used (see [1], pp. 264-266, 294). In the isotropic case

$$\begin{aligned} \Phi &= a\mathbf{q}^2 + b_\alpha \dot{\eta}_\alpha^2 + 2c_\alpha \mathbf{q} \cdot \dot{\eta}_\alpha, \quad -\nabla P = \frac{1}{2} \frac{\partial \Phi}{\partial \mathbf{q}} = a\mathbf{q} + c_\alpha \dot{\eta}_\alpha, \\ -\mathbf{B}_\alpha &= \frac{1}{2} \frac{\partial \Phi}{\partial \dot{\eta}_\alpha} = c_\alpha \mathbf{q} + b_\alpha \dot{\eta}_\alpha. \end{aligned} \quad (7)$$

In the last equation there is no summation over  $\alpha$ .

To establish a direct relation between  $\nabla P$  and  $\mathbf{q}$ , the parameters  $\eta_\alpha$  must be eliminated from Eq. (7). It is sufficient to use an approximation quadratic in  $\eta_\alpha$  for the function  $F(\eta_\alpha) = U - TS$ , and to pass to the Laplace transform in Eq. (7) [4]

$$U = TS + \frac{\lambda_{\alpha\beta}}{2} \eta_\alpha \cdot \eta_\beta.$$

Hence, when Eq. (2) is taken into account,  $\mathbf{B}_\alpha = \lambda_{\alpha\beta} \eta_\beta$  and Eq. (7) may be written in transforms as follows

$$-\nabla P = a\mathbf{q} + sc_\alpha \eta_\alpha, \quad -\lambda_{\alpha\beta} \eta_\beta = c_\alpha \mathbf{q} + sb_\alpha \eta_\alpha. \quad (8)$$

Filtration is assumed to begin at time  $t = 0$  from a state of rest.

The positive definite matrix  $\Lambda$  with elements  $\lambda_{\alpha\beta}$  and the diagonal matrix  $B$  with elements  $b_\alpha$  may be written at the same time in the form (see [5], p. 80)

$$\Lambda = L'L, \quad B = L'T_0L,$$

where  $L$  is a nondegenerate matrix;  $T_0$  is a diagonal matrix with elements  $\tau_\alpha > 0$ . This allows the solution of the second relation in Eq. (8) to be written in the form

$$\eta_i = -\frac{m_{\alpha i}}{1 + \tau_\alpha s} m_{\alpha\beta} c_\beta \mathbf{q} \quad (9)$$

( $m_{ik}$  are elements of the matrix  $M = L^{-1}$ ).

Substituting Eq. (9) into the first relation of Eq. (8), a direct relation is obtained between the pressure gradient and the filtration velocity in terms of transforms

$$-\nabla P = \left( a - \frac{\mu_\alpha}{\tau_\alpha} \right) \mathbf{q} + \left( \frac{\mu_\alpha}{\tau_\alpha} \cdot \frac{1}{1 + \tau_\alpha s} \right) \mathbf{q}, \quad \mu_\alpha = (c_i m_{i\alpha})^2.$$

Inverting the transforms gives

$$-\nabla P = \left( a - \frac{\mu_\alpha}{\tau_\alpha} \right) \mathbf{q} + \int_0^t \left( \frac{\mu_\alpha}{\tau_\alpha} \exp[-(t-\xi)/\tau_\alpha] \right) \mathbf{q}(\xi) d\xi. \quad (10)$$

In the particular case  $n = 1$ , Eq. (10) is equivalent to the following relation [6]

$$-\left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \nabla P = a \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) \mathbf{q}.$$

The constants  $\lambda_1$  and  $\lambda_2$  are expressed in terms of factors of quadratic form  $\Phi$  and F

$$\lambda_1 = \frac{b_1}{\lambda_{11}}, \quad \lambda_2 = \frac{ab_1 - c_1^2}{\lambda_{11}a}.$$

Here  $\lambda_2/\lambda_1 > 1$ .

#### NOTATION

$Q(e)$ , external heat flux;  $Q'$ , uncompensated heat;  $U$ , interval energy;  $S$ , entropy;  $T$ , temperature;  $dA^{(i)}$ , elementary work of the internal surface forces;  $\eta_\alpha$ , internal degrees of freedom;  $B_\alpha$ , thermodynamic parameters conjugate to  $\eta_\alpha$ ;  $\Phi$ , dissipative function;  $\sigma_{ij}$ , stress tensor in the liquid;  $v_i$ , liquid-particle velocity;  $V_0$ , volume of the porous medium;  $V$ , liquid volumes;  $S_0$ , surface of volume  $V_0$ ;  $S_e$ , "liquid part" of  $S_0$ ;  $S_i$ , liquid contact surface with the porous matrix inside  $V_0$ ;  $P$ , mean (over  $S_e$ ) liquid pressure;  $q_i$ , filtration velocity;  $s$ , Laplace-transform parameters;  $t$ , time.

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#### COMPONENT-TRANSFER EQUATION IN COLUMN WITH LONGITUDINAL SEPARATION

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By the method of "integration over the transverse coordinate," a transfer equation in a separating column is obtained, taking the longitudinal enrichment mechanism into account.

The most widespread type of equipment for isotope separation in gas or liquid phases is a column in which the transverse enrichment effect is converted into a longitudinal effect as a result of circulatory flow of the mixture [1]. The derivation of the transfer equation describing the axial distribution of the mean (over the column cross section) concentration of liberated isotope is based on the equilibrium conditions for the component fluxes and the total flux of mixture. To calculate the separation characteristics of equipment of the usual type (thermodiffusional [2, 3] or mass-diffusional [4] columns), it is sufficient to use the transfer equation taking only the radial separation process into account. This kind of relation is obtained, e.g., in [1] by the approximate method of integration over the transverse coordinate [5]. At the same time, in some equipment (in particular, in a separating system with a traveling magnetic wave [6-8]), in addition to the transverse enrichment effect, intensified by forced countercurrent motion of the mixture, there is also a primary longitudinal separation process. There then arises a situation in which circulation has a double effect on the distribution since, on the one hand, it facilitates the multiplication of the transverse effect over the length of the column and, on the other, it causes agitation and, correspondingly, a reduction in the primary longitudinal effect. Below, on the basis of the method of calculations outlined in [1], a transfer equation in a column when both transverse and longitudinal enrichment mechanisms are present is obtained.

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